

Notes on Local Grand Unification*

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Abstract

Grand unified models in four dimensions typically suffer from the doublet-triplet splitting problem. This obstacle can be overcome in higher-dimensional settings, where a non-trivial gauge group topography can explain the simultaneous appearance of complete standard model generations in the form of **16**-plets of $SO(10)$ and the Higgs fields as split multiplets. In these notes, the emerging scheme of ‘local grand unification’ and its realization in the context of orbifold compactifications of the heterotic string are reviewed.

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1 Introduction

The standard model (SM) is remarkably successful in explaining observations. It is based on the gauge group

$$G_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y , \quad (1)$$

and has three generations of matter transforming as

$$(\mathbf{3}, \mathbf{2})_{1/6} + (\bar{\mathbf{3}}, \mathbf{1})_{-2/3} + (\bar{\mathbf{3}}, \mathbf{1})_{1/3} + (\mathbf{1}, \mathbf{2})_{-1/2} + (\mathbf{1}, \mathbf{1})_1 . \quad (2)$$

It further contains the Higgs field which transforms as $(\mathbf{1}, \mathbf{2})_{1/2}$. The evidence for neutrino masses strongly supports the existence of right-handed neutrinos, which are singlets under G_{SM} . One of the features of SM matter that points towards a deeper underlying structure is charge quantization, i.e. the fact that hypercharges of different multiplets are integer multiples of a common unit. The most compelling explanation of this fact arises in schemes where hypercharge is embedded in a non-Abelian group factor [1], in particular in grand unified theories (GUTs) [2]. In the context of an $\text{SU}(5)$ GUT, the five irreducible representations of (2) can be combined into two $\text{SU}(5)$ representations [2],

$$\mathbf{10} = (\mathbf{3}, \mathbf{2})_{1/6} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-2/3} \oplus (\mathbf{1}, \mathbf{1})_1 , \quad (3a)$$

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{1/3} \oplus (\mathbf{1}, \mathbf{2})_{-1/2} . \quad (3b)$$

That is, one generation of standard model fermions (without the right-handed neutrino) can be combined into two irreducible representations of $\text{SU}(5)$. Hypercharge is in the Cartan basis of $\text{SU}(5)$ but not in the Cartan bases of $\text{SU}(3)_C$ or $\text{SU}(2)_L$.

Even more impressive than the $\text{SU}(5)$ relations (3) is the fact that a **16**-plet of $\text{SO}(10)$ comprises one full SM generation including the right-handed neutrino [3, 4],

$$\mathbf{16} = (\mathbf{3}, \mathbf{2})_{1/6} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-2/3} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{1/3} \oplus (\mathbf{1}, \mathbf{2})_{-1/2} \oplus (\mathbf{1}, \mathbf{1})_1 \oplus (\mathbf{1}, \mathbf{1})_0 . \quad (4)$$

While charge quantization can also be attributed to the requirement of anomaly cancellation, the scheme of GUTs receives further support from gauge coupling unification. It is well known that the gauge couplings unify in the minimal supersymmetric extensions of the standard model (MSSM) [5] at the scale

$$M_{\text{GUT}} \simeq 2 \cdot 10^{16} \text{ GeV} . \quad (5)$$

In these notes, we will take this observation seriously and from now on consider theories with low-energy supersymmetry that are consistent with MSSM gauge coupling unification.

Unfortunately, the SM Higgs sector casts some shadow on the standard picture of GUTs. The smallest $\text{SO}(10)$ representation containing the MSSM Higgs doublets (or the SM Higgs) is the **10**-plet, which decomposes as

$$\mathbf{10} = (\mathbf{1}, \mathbf{2})_{1/2} \oplus (\mathbf{1}, \mathbf{2})_{-1/2} \oplus (\mathbf{3}, \mathbf{1})_{-1/3} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{1/3} . \quad (6)$$

While $SU(2)$ doublets are required for electroweak symmetry breaking, there are strong experimental constraints on the triplet masses. They come from effective dimension 5 operators [6, 7, 8] that arise from integrating out the triplets and lead to proton decay (cf. [9]); see figure 1. This leads, first of all, to the so-called doublet-triplet splitting problem, which can be phrased as:

“Why does matter appear in complete GUT representations while the Higgs multiplets are incomplete (split) ?”

Note that the problem has two aspects. On the one hand, in the context of grand unification one might ask why the Higgses are split, on the other hand in a theory without unification one would like to understand why matter apparently fits into representations of a larger group.

Although there have been ingenious proposals to solve the problem in the context of four-dimensional (4D) grand unification (see e.g. [10, 11]), it is probably fair to say that none of these renders the theory as simple and beautiful as the original idea of GUTs. Besides, these solutions give a GUT scale mass to the triplets, which might not be enough to suppress the dimension 5 proton decay operators [9].

2 Orbifold GUTs

Arguably, the most appealing solutions to the doublet-triplet splitting problem are obtained in higher-dimensional extensions of the standard model. While this point has been stressed from early on in the string literature (cf. [12, 13]), in these notes we will use the framework of orbifold GUTs [14, 15, 16, 17, 18, 19, 20] (for a review, see e.g. [21]) in order to see how this works. Later, in sections 3 and 4, these constructions will be embedded into string theory.

From now on, let us entertain the possibility that the known four space time dimensions are amended by extra compact dimensions. That is, we consider gauge theories, based on the gauge group G , on the space

$$M^4 \times K, \tag{7}$$

where M^4 is the usual Minkowski space and K is compact. There are some (phenomenological) constraints on the properties of the internal space K . The most important requirement for the subsequent discussion is that K be such as to give rise to a chiral spectrum in 4D effective field theory which emerges at scales below the compactification scale.

2.1 One-dimensional orbifold GUTs

The simplest setting satisfying this requirement is an S^1/\mathbb{Z}_2 orbifold (figure 2). An orbifold, in general, emerges from dividing a manifold by one of its (non-freely acting) discrete symmetries. In the S^1/\mathbb{Z}_2 case one first compactifies one extra dimension on a circle and then identifies points related by a \mathbb{Z}_2 reflection symmetry, which is a symmetry of the circle. The emerging

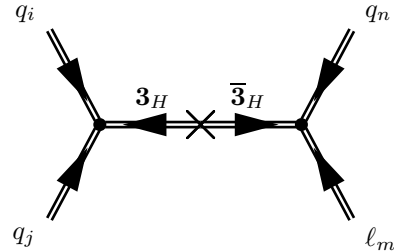


Figure 1: Proton decay via triplet exchange.

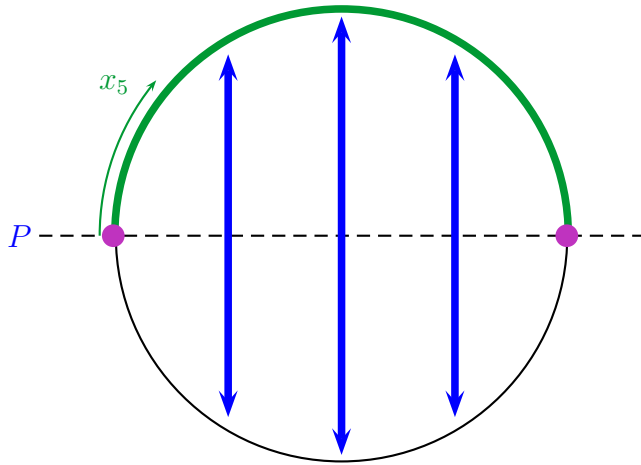


Figure 2: S^1/\mathbb{Z}_2 orbifold.

space, that is the fundamental domain of the orbifold, is an interval which is bounded by the orbifold fixed points, i.e. the two points that are invariant under the orbifold action.

The important point in the context of GUT model building is that one can associate an automorphism of the gauge group with the reflection in the internal space. That is, the orbifold action is

$$\begin{aligned} \Theta &: x_5 \rightarrow \theta x_5 = -x_5, \\ &\mathbf{r} \rightarrow P_{\mathbf{r}} \mathbf{r}. \end{aligned} \quad (8)$$

Here, \mathbf{r} is a representation of G and $P_{\mathbf{r}}$ is the representation (matrix) of the automorphism. Since P squares to the identity in internal space, we will require that $R_{\mathbf{r}}(P)^2 = \mathbb{1}$.¹

The most important application of the embedding of the orbifold action in the gauge degrees of freedom is gauge symmetry breaking by orbifolding. Consider $M^4 \times S^1/\mathbb{Z}_2$ and a state transforming under the representation \mathbf{r} . The requirement that the state be invariant under the orbifold action amounts to

$$\phi_{\mathbf{r}}(x_0, \dots, x_3; x_5) = P_{\mathbf{r}} \phi_{\mathbf{r}}(x_0, \dots, x_3; -x_5). \quad (9)$$

In the \mathbb{Z}_2 case under consideration this leads to various “orbifold parities”.

Consider, as an example, the orbifold S^1/\mathbb{Z}_2 with gauge group $SU(3)$. In this example, let us focus on a non-supersymmetric setting. For the automorphism in the fundamental representation take

$$P = \text{diag}(-1, -1, 1). \quad (10)$$

This means that scalar bulk fields transforming as **3**-plets under $SU(3)$ have to satisfy

$$\phi(x_{\mu}, -x_5) = P \phi(x_{\mu}, x_5). \quad (11)$$

¹This requirement might potentially be relaxed, see [22, 23].

This implies the boundary condition

$$\phi(x, 0) = P \phi(x, 0) \quad (12)$$

on the interval. Since $x_5 = L$ and $x_5 = -L$ coincide, one also has

$$\phi(x, L) = P \phi(x, L) . \quad (13)$$

Let us now work with the eigenstates of P . The field ϕ decomposes into two pieces, the eigenstates of P with eigenvalues $+1$ and -1 , which we will denote by ϕ_{\pm} , respectively. In this notation, equation (11) reads

$$\phi_{\pm}(x_{\mu}, -x_5) = P \phi_{\pm}(x_{\mu}, x_5) = \pm \phi_{\pm}(x_{\mu}, x_5) . \quad (14)$$

Since the fifth dimension is compact, we can expand the eigenstates in terms of Fourier modes [14],

$$\phi_{+}(x_{\mu}, x_5) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\delta_{n0}\pi} R} \phi_{+}^{(n)}(x_{\mu}) \cos\left(\frac{n x_5}{R}\right) , \quad (15a)$$

$$\phi_{-}(x_{\mu}, x_5) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi} R} \phi_{-}^{(n)}(x_{\mu}) \sin\left(\frac{n x_5}{R}\right) . \quad (15b)$$

Here R is the radius of the \mathbb{S}^1 and the length of the interval is therefore $L = R\pi$. In the 4-dimensional effective theory, the Fourier modes can be regarded as massive states with masses n/R . In particular, only the ϕ_{+} field possesses a zero-mode. This implies that at energies below $1/R$ one can integrate out the heavy modes, thus obtaining an effective theory with only the ϕ_{+} zero-modes as dynamical degrees of freedom.

An analogous analysis can be carried out for the gauge fields. Here, the boundary conditions for the four-dimensional vector fields read

$$A_{\mu}^a(x_{\mu}, -x_5) T_a = A_{\mu}^a(x_{\mu}, x_5) P T_a P^{-1} , \quad (16)$$

with T_a denoting the generators. It is straightforward to check that only the gauge bosons of an $SU(2) \times U(1)$ subgroup of the original $SU(3)$ have zero-modes in 4D. For instance, a simple explicit calculation with the standard $SU(3)$ matrices (see e.g. [24, p. 502]) reveals that only the gauge bosons associated with $\lambda_i/2$ where $i \in \{1, 2, 3, 8\}$ survive the projection condition (16). The zero-mode of $\phi_{+}^{(0)}$ transforms as a singlet under the low-energy gauge group $SU(2) \times U(1)$.

This example illustrates some important aspects of orbifold compactifications:

- (i) Gauge symmetry gets reduced.
- (ii) Bulk fields furnishing representations under the (larger) bulk gauge symmetry survive the projection conditions only partially. That means that the same mechanism that breaks the gauge symmetry leads to the appearance of split multiplets.

The simplest setup highlighting the second point is the Kawamura model [14, 15].

Kawamura proposed an orbifold compactification with bulk gauge symmetry $SU(5)$. As we have already seen, an S^1/\mathbb{Z}_2 orbifold compactification can be equivalently described as a (field) theory on an interval whereby the boundary conditions, imposed at the ends of the interval, can involve an automorphism of the gauge group. In the original papers [14, 15, 25], orbifolds $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ were constructed in order to allow for different boundary conditions at the ends of the interval. In these notes, we will not describe this mechanism in detail since the appearance of different boundary conditions at different fixed points can be attributed to the presence of discrete Wilson lines, as we shall see later (see subsection 3.2). The important point is that either way one arrives at a setup where the boundary conditions at $x_5 = 0$ for bulk fields in the fundamental representation read

$$A_\mu^a(x_\mu, x_5 = 0) T_a = A_\mu^a(x_\mu, x_5 = 0) P T_a P^{-1}, \quad (17a)$$

$$\phi(x_\mu, x_5 = 0) = P \phi(x_\mu, x_5 = 0), \quad (17b)$$

and at $x_5 = L$

$$A_\mu^a(x_\mu, x_5 = L) T_a = A_\mu^a(x_\mu, x_5 = L) P' T_a P'^{-1}, \quad (18a)$$

$$\phi(x_\mu, x_5 = L) = P' \phi(x_\mu, x_5 = L). \quad (18b)$$

Here, we assumed that ϕ transforms as a **5**-plet.

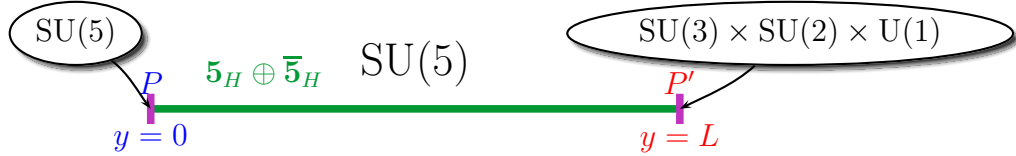


Figure 3: One-dimensional orbifold with non-trivial gauge group topography.

Denoting the eigenstates of the projections P and P' by $\phi_{\pm\pm}$, we can expand these fields in terms of Fourier eigenstates (cf. [25]),

$$\phi_{++}(x_\mu, x_5) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n0}} \pi R}} \phi_{++}^{(2n)}(x_\mu) \cos\left(\frac{2n x_5}{R}\right), \quad (19a)$$

$$\phi_{+-}(x_\mu, x_5) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{+-}^{(2n+1)}(x_\mu) \cos\left(\frac{(2n+1) x_5}{R}\right), \quad (19b)$$

$$\phi_{-+}(x_\mu, x_5) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{-+}^{(2n+1)}(x_\mu) \sin\left(\frac{(2n+1) x_5}{R}\right), \quad (19c)$$

$$\phi_{--}(x_\mu, x_5) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{--}^{(2n+2)}(x_\mu) \sin\left(\frac{(2n+2) x_5}{R}\right). \quad (19d)$$

The only state that leads to a 4D zero-mode is ϕ_{++} . That is, 4D zero-modes are the modes that survive all projection conditions simultaneously.

The bulk states of the Kawamura model are the gauge bosons of $SU(5)$ as well as $\mathbf{5}$ and $\bar{\mathbf{5}}$ hypermultiplets. The fact that the latter are hypermultiplets means that in 4D language each of these gives rise to two chiral multiplets (cf. e.g. [26]). The gauge multiplet in 5D gives rise to the usual 4D gauge fields plus a 4D chiral superfield.

For $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$, P can be represented by the matrix $P = \text{diag}(1, 1, 1, 1, 1) =: \text{diag}(+, +, +, +, +)$ and P' by $P' = \text{diag}(+, +, -, -, -)$.² The action of P' is the one that breaks $SU(5)$. One easily confirms that the gauge theory on the P' -brane is reduced to $SU(3) \times SU(2) \times U(1)$. Apart from that, starting with a $\mathbf{5}$ -plet in the bulk, one can repeat the analysis of the previous example. The projection conditions are such that in the decomposition

$$\mathbf{5}_{\text{hyper}} \rightarrow [(\mathbf{3}, \mathbf{1})_{-1/3} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{1/3} \oplus (\mathbf{1}, \mathbf{2})_{1/2} \oplus (\mathbf{1}, \mathbf{2})_{-1/2}]_{\text{chiral}} \quad (20)$$

under $SU(5)$ in 5D to G_{SM} in 4D only the doublet $(\mathbf{1}, \mathbf{2})_{1/2}$ has a zero-mode. The other modes, in particular all triplets, get heavy. The same mechanism that is used for gauge symmetry breaking leads to doublet-triplet splitting!

Of course, in order to obtain a potentially realistic model, one also has to include the SM matter. In the original Kawamura model [15] (and some of its variations [16, 17]), SM matter states are brane fields living at the $SU(5)$ brane $x_5 = 0$. It is clear that at this point matter has to appear in complete representations of the local, i.e. $SU(5)$, gauge symmetry. Simply stated, fields living at $x_5 = 0$ ‘do not care’ for what’s going on at $x_5 = L$.

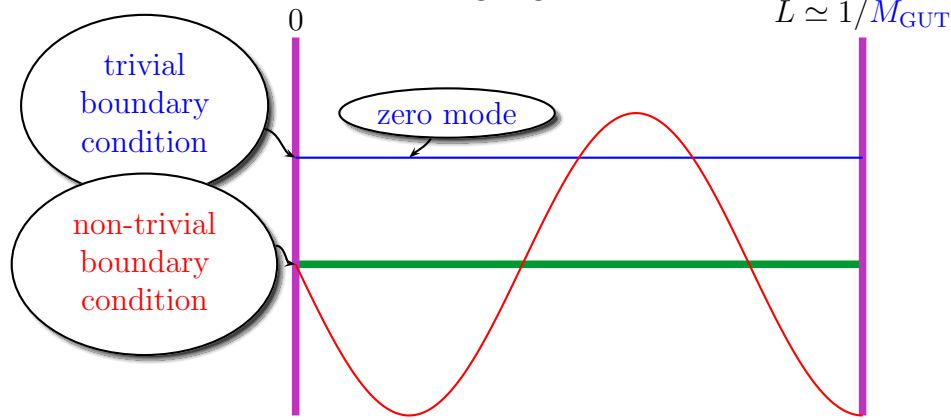


Figure 4: Gauge symmetry breaking by boundary conditions.

What one can learn from the example is that higher-dimensional models offer an intuitive explanation of the presence of complete and split multiplets at the same time.

Furthermore, it has been shown that the dimension 5 proton decay operators, which usually lead to problems in 4D SUSY GUTs, are naturally absent in higher dimensions. If the Higgs multiplets live in the bulk, the mass partners of the triplets do not couple to SM matter, and therefore the triplet exchange diagram does not exist [16, 17]. Another possibility (which, however, ruins the intuitive explanation of SM matter in terms of complete $SU(5)$ representations) is to introduce the Higgs on the G_{SM} brane at $x_5 = L$. Here, it is not necessary to introduce the

²Note that with this convention, P' and $-P'$ act the same way on the adjoint representation, $\text{ad } SU(5) \sim (\mathbf{5} \times \bar{\mathbf{5}})_{\text{traceless}}$. That is, the action of P' on the adjoint can be represented by an inner automorphism.

triplet at all, and therefore the diagram does not exist either [18]. Although similar suppression mechanisms exist in 4D models (cf. [27]), it is intriguing that the dimension 5 proton decay problem can be solved so easily in higher-dimensional settings.

There are further lessons to be learned by going beyond 5D.

2.2 Higher-dimensional orbifolds

Let us now turn to the discussion of higher-dimensional, field-theoretic orbifolds. At this point, it is advantageous to introduce the usual orbifold language [28, 29]. We are interested in orbifolds that emerge from dividing a d -dimensional torus \mathbb{T}^d by one of its symmetries. The torus \mathbb{T}^d can be understood as \mathbb{R}^d/Γ , i.e. as the d -dimensional space with points differing by lattice vectors $e_\alpha \in \Gamma$ identified. The lattice Γ respects a reflection symmetry, which maps Γ onto itself. We will often be interested in cases where the lattice enjoys discrete rotational symmetries, i.e. there exists an automorphism $\theta \in O(d)$ of the lattice with $\theta^N = \mathbb{1}$ for some $N \in \mathbb{N}$. The set of all such symmetries forms the point group \mathcal{P} . We will be interested in a \mathbb{Z}_N orbifold, where any element of the point group can be written as θ^k with $0 \leq k \leq N-1$ (N is the smallest integer with $\theta^N = \mathbb{1}$). One can define an orbifold as the quotient $\mathbb{O} = \mathbb{T}^d/\mathcal{P}$. Equivalently one can describe the orbifold by

$$\mathbb{O} = \frac{\mathbb{R}^d}{\mathcal{S}}, \quad (21)$$

where the space group \mathcal{S} comprises of the elements of the point group and lattice translations,

$$\mathcal{S} = \{g; g = (\theta^k, n_\alpha e_\alpha)\}. \quad (22)$$

Space group elements $g \in \mathcal{S}$ act on the coordinates of the compact space as

$$g x = \theta^k x + n_\alpha e_\alpha. \quad (23)$$

Two elements $g, h \in \mathcal{S}$ can be multiplied according to the rule

$$g \cdot h = (\theta^k, n_\alpha e_\alpha) \cdot (\theta^\ell, m_\alpha e_\alpha) = (\theta^{k+\ell}, n_\alpha e_\alpha + \theta^k m_\alpha e_\alpha). \quad (24)$$

Let us first consider the orbifold $\mathbb{T}^2/\mathbb{Z}_2$, which emerges from dividing the torus \mathbb{T}^2 by a point reflection symmetry. The torus is defined by 2 (linearly independent) lattice vectors e_1 and e_2 spanning the fundamental domain (in figure 5 the fundamental domain is given by the two shaded regions). The \mathbb{Z}_2 then acts as a reflection (or, equivalently, a 180° rotation represented by the purple arc) about an arbitrary lattice node which one could call the ‘origin’. Certain points are mapped under the orbifold action onto themselves (up to lattice translations); these are the orbifold fixed points (red bullets). By identifying points in the fundamental domain of the torus, that are related by the \mathbb{Z}_2 orbifold action, one arrives at the fundamental domain of the orbifold. In figure 5 this is the darker shaded region. By folding the fundamental domain along the line connecting the upper two fixed points and gluing the adjacent edges together, one arrives at a ravioli- (cf. [30]) or pillow- (cf. [22]) like object which is smooth everywhere except for the corners, i.e. the orbifold fixed points.

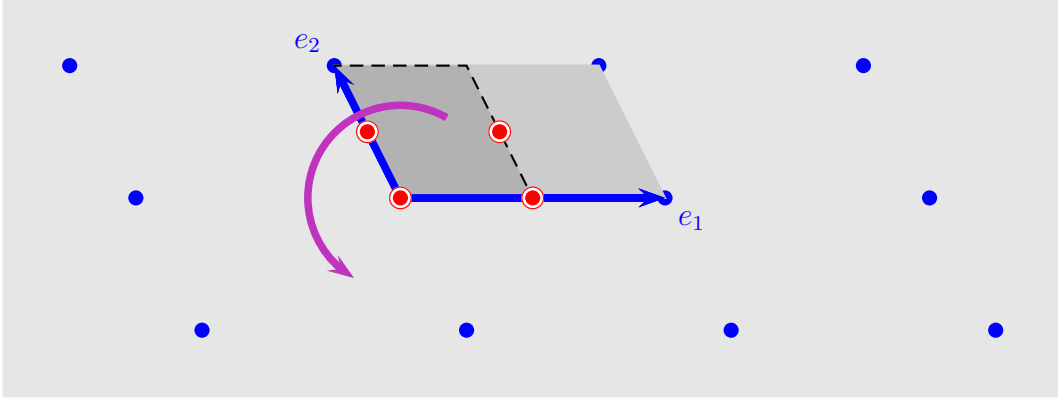


Figure 5: $\mathbb{T}^2/\mathbb{Z}_2$ orbifold.

As a specific example of a 6D orbifold, let us discuss the Asaka-Buchmüller-Covi model [19, 31]. Here, the gauge embedding (‘orbifold parities’) is chosen such that $\text{SO}(10)$ is broken to $G_{\text{GG}} = \text{SU}(5) \times \text{U}(1)$, the Pati-Salam symmetry G_{PS} and the so-called flipped $\text{SU}(5)$ G_{fl} at three of the four fixed points while the gauge embedding is trivial at the fourth fixed point (cf. figure 2.2). It is a remarkable (group-theoretic) fact that the intersection of G_{GG} and G_{PS} in $\text{SO}(10)$ yields the standard model, G_{SM} , up to a $\text{U}(1)$ factor. In order to accommodate the observed three generations, the authors of [19, 31] placed one generation at each of the G_{GG} , G_{PS} and G_{fl} fixed points. Clearly, those localized fields have to furnish complete representations of the ‘local’ gauge groups realized at the fixed points. The Higgs arises from the bulk **10**-plets whereby only the electroweak doublets survive all projection conditions and have a zero-mode.

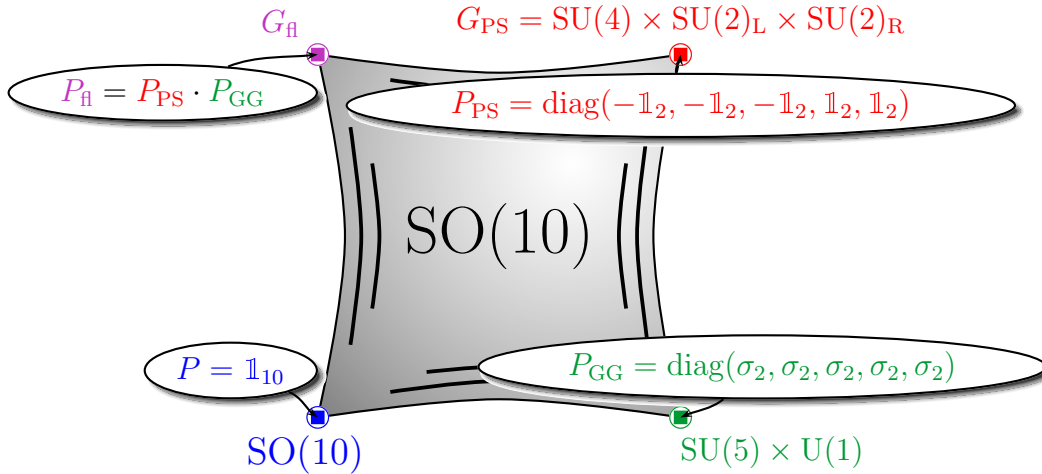


Figure 6: Asaka-Buchmüller-Covi model.

There are certain lessons that can be learned from 2-dimensional orbifolds:

1. these constructions can exhibit a ‘non-trivial gauge group topography’, i.e. a bulk group gets broken to different ‘local groups’ at different fixed points;

2. the low-energy gauge group is the intersection of the local groups in the bulk group;
3. localized matter comes in complete representations of the local gauge groups;
4. bulk fields get split, i.e. are partially projected out.

Nevertheless, it is also clear that these models leave many questions unanswered. One would, for instance, like to obtain a deeper understanding of the field content, i.e. the bulk fields and the states localized at the fixed points. Further, it would be useful to have a theory behind the couplings among various fields. It is also clear that calculations based on higher-dimensional field theories (which are known to be non-renormalizable) cannot be fully trusted and require a UV completion. Gravity should also be incorporated. Last but not least, the orbifold GUTs in the literature provide a nice explanation of the fact that the Higgses appear as split multiplets, however the fact that matter generations fit into **16**-plets of $SO(10)$ has no convincing explanation. In what follows, we will explain how string-derived orbifolds provide a framework to address all these questions.

3 Orbifold compactifications of the heterotic string

Orbifold compactifications of the heterotic string have a long history [28, 29, 32, 33, 34, 35, 36, 37, 38, 39] and remain the most phenomenologically attractive string framework. As is well-known, there are 5 string theories. Since we aim at obtaining **16**-plets of $SO(10)$ (at the perturbative level) as well as gauge unification, the heterotic string [40, 41] is singled out. A natural setup accommodating these features is the $E_8 \times E_8$ heterotic string, which will be our focus in what follows. Introductory lectures on $E_8 \times E_8$ heterotic orbifolds can be found in [42].

The first string-derived orbifold GUTs were presented in [43, 44, 45, 46]. While in [44] the stringy calculation of the spectrum and the connection between Wilson lines and non-trivial gauge group topographies are explained in great detail, Ref. [43, 45] provides an orbifold GUT interpretation of heterotic orbifolds and derives models with Pati-Salam gauge symmetry and three chiral generations of matter. Further, Ref. [46] presents an MSSM-like model with 3 localized **16**-plets and studies its orbifold GUT limits in various (5 to 9) dimensions. These models are based on a \mathbb{Z}_6 -II orbifold, which we describe in the next subsection.

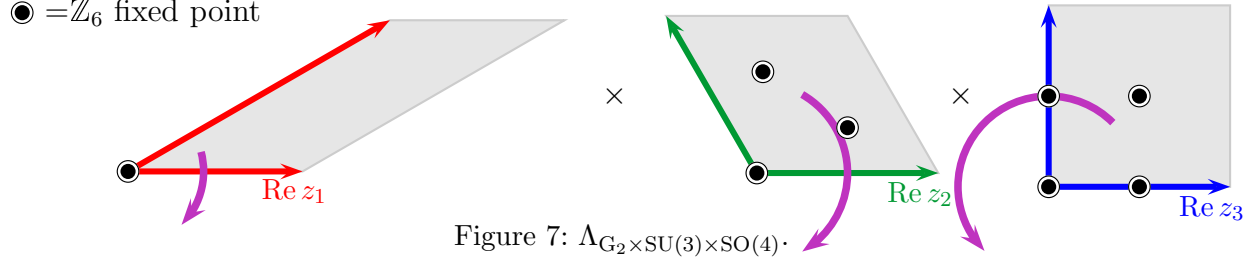
3.1 Orbifold basics (using the \mathbb{Z}_6 -II orbifold)

The compactification lattice can be chosen as (see figure 7)

$$\Lambda_{G_2 \times SU(3) \times SO(4)} := \text{root lattice of Lie algebra of } G_2 \times SU(3) \times SO(4) . \quad (25)$$

The \mathbb{Z}_6 action is a -60° rotation in the G_2 plane, a -120° rotation in the $SU(3)$ plane and a 180° rotation (or reflection) in the $SO(4)$ plane. Parametrizing the three 2-tori by complex coordinates z_i ($1 \leq i \leq 3$), the orbifold action on the compact six dimensions reads

$$z_i \rightarrow e^{2\pi i v_6^i} z_i \quad \text{with} \quad v_6 = \frac{1}{6}(-1, -2, 3) . \quad (26)$$



This orbifold twist is accompanied by the corresponding action on the gauge degrees of freedom. Using the string notation, one has (cf. [44, 47])

$$(\theta, 0) : X^I \rightarrow X^I + \pi V_6^I, \quad (27)$$

where $(\theta, 0) \in \mathcal{S}$, $6 V_6 \in \Lambda_{E_8 \times E_8}$ for the action to be \mathbb{Z}_6 . Here, X^I ($1 \leq I \leq 16$) denote the left-moving string coordinates which are compactified on the $E_8 \times E_8$ torus $\Lambda_{E_8 \times E_8}$. Moreover, the torus translations can be associated with the so-called Wilson lines [32], e.g.

$$(\mathbb{1}, e_5) : z_3 \rightarrow z_3 + 1 \leftrightarrow X^I \rightarrow X^I + \pi W_2, \quad (28)$$

where $2 W_2 \in \Lambda_{E_8 \times E_8}$. Wilson lines are subject to certain constraints; the \mathbb{Z}_6 -II orbifold discussed here allows for two Wilson lines of order 2 associated with the two independent translations in the $SO(4)$ torus and one Wilson line of order 3 in the $SU(3)$ torus ($3 W_3 \in \Lambda_{E_8 \times E_8}$).

3.2 Orbifold construction kit

The presence of Wilson lines leads to the picture where, as in the 6D orbifolds, the bulk gauge group gets broken to different subgroups at different fixed points. Each symmetry breakdown can be attributed to the *local gauge shift* of the form

$$V_{\text{local}} = V_6 + \text{Wilson lines} \leftrightarrow G_{\text{local}}. \quad (29)$$

Here, the local gauge group G_{local} corresponds to the gauge bosons which fulfill $p \cdot V_{\text{local}} \in \mathbb{Z}$ (see [47] for details). Gauge interactions surviving compactification are mediated by the gauge bosons that satisfy all projection conditions simultaneously. In other words, they correspond to the intersection of all local gauge groups in the bulk group. This leads to an alternative way of constructing orbifolds with Wilson lines: start with a local shift, i.e. with a ‘corner’ of the fundamental domain, and glue it to the other corners. The result is a ‘pillow’ with non-trivial gauge group topography (figure 8). It is important that, due to stringy constraints, this procedure does not allow for arbitrary gauge groups at different corners. In particular, modular invariance [48], which guarantees anomaly freedom, restricts possible choices of the gauge shifts and Wilson lines.

3.3 Space group versus fixed points

Space group elements enjoy the group multiplication law (24). Hence, they come in equivalence classes. Two space group elements g and g' are in the same class if there is an element $h \in \mathcal{S}$

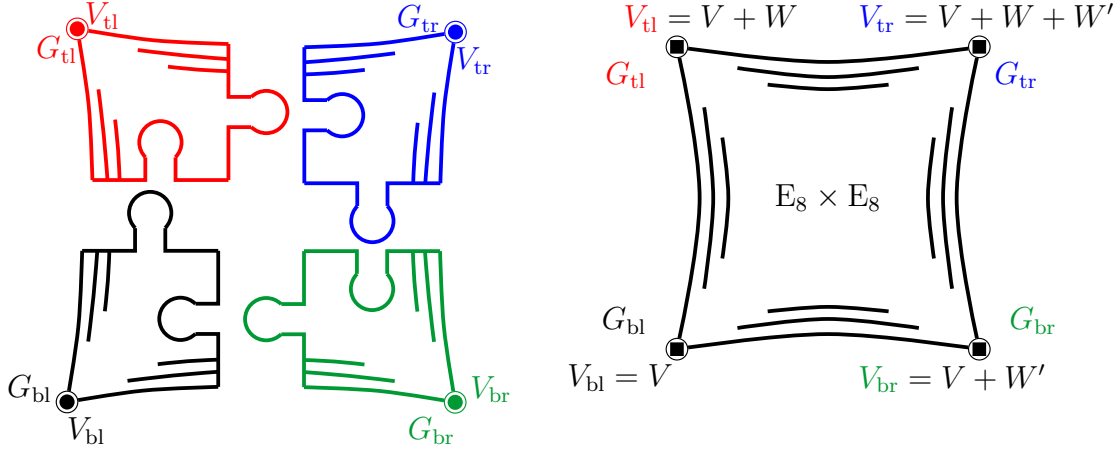


Figure 8: A 2D sketch of the orbifold construction kit. The gauge group after compactification is $G_{\text{low-energy}} = G_{\text{bl}} \cap G_{\text{tl}} \cap G_{\text{br}} \cap G_{\text{tr}}$.

such that

$$g = h g' h^{-1}. \quad (30)$$

Each conjugacy class corresponds to one fixed point in the fundamental domain. In non-prime orbifolds such as the \mathbb{Z}_6 -II orbifold, the situation is more complicated since there are also fixed planes (see [47] for details).

3.4 Spectrum

We are interested in the zero-modes on orbifolds. Each state can be associated with a conjugacy class, or a space group element $(\theta^k, n_\alpha e_\alpha)$, which will be referred to as the ‘constructing element’. The (zero-mode) spectrum decomposes into an untwisted ($k = 0$) and various twisted ($k \neq 0$) sectors. All massless states correspond to closed strings, while twisted states correspond to strings that are closed only upon orbifolding (figure 9(a)). One associates

$$\text{massless state} \leftrightarrow \text{constructing element } (\theta^k, n_\alpha e_\alpha) \leftrightarrow \begin{cases} \text{fixed point} & : k = 1, 5, \\ \text{fixed plane} & : k = 2, 3, 4, \\ \text{bulk} & : k = 0. \end{cases}$$

This amounts to a dictionary between constructing elements and localization properties of the corresponding state in the field-theoretic language (figure 9(b)). Given the geometry and gauge embedding (i.e. the gauge shift and Wilson lines), one can calculate the spectrum following a straightforward procedure (cf. [44]). For the \mathbb{Z}_6 -II calculation, see [45, 47].

3.5 Selection rules

Given the field content of the orbifold, the next step is to study interactions of the theory. Couplings on orbifolds are governed by certain selection rules [49, 50]. From the effective field theory perspective, these rules are

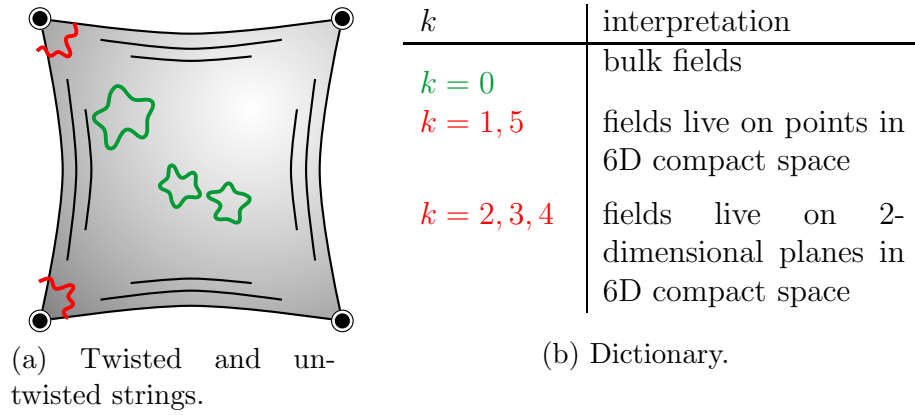


Figure 9: Bulk and brane fields from untwisted and twisted strings.

- gauge invariance,
- (non-Abelian) discrete symmetries, and
- discrete R -symmetries.

The selection rules for the \mathbb{Z}_6 -II orbifold are summarized in [47] (a careful discussion of the space-group rule is presented in [51]).

3.6 Summary of orbifold basics

The main lesson is that, once the geometry and gauge embedding are fixed, the theory is completely determined. In particular, unlike in the field-theoretic constructions, the spectrum is calculable and one cannot ‘invent’ representations and/or couplings.

4 Local grand unification

Having discussed how to compactify the heterotic string on an orbifold, let us now turn to the applications – construction of potentially realistic string models.

4.1 First string-derived orbifold GUT

The first constructions where the intuition gained in orbifold GUTs was used to choose appropriate geometry and gauge embeddings were the Pati-Salam models by Kobayashi, Raby and Zhang (KRZ) [43, 45]. KRZ constructed a model with the following interesting features:

- gauge group upon compactification is G_{PS} ;
- spectrum is 3 generations plus vector-like matter under G_{PS} ;

- Pati-Salam Higgses, necessary to break G_{PS} to G_{SM} , are present in the spectrum;
- two generations live at fixed points with an $\text{SO}(10)$ GUT symmetry;
- many exotics can be given masses by assigning vacuum expectation values (vevs) to Pati-Salam singlets.

In addition to these positive features, there are a number of shortcomings. Perhaps, the main issue is the breaking of the Pati-Salam gauge symmetry to that of the Standard Model. In string theory one cannot write down an arbitrary Higgs potential – it must be derived. In particular, it must be consistent with various selection rules (as well as supersymmetry).

Some of these problems can be avoided by breaking the 10D gauge symmetry directly to that of the SM (times extra factors) upon compactification. This strategy allows one to construct many models with the exact MSSM spectrum while keeping nice features of GUTs.

4.2 Local grand unification

The successful aspects of the KRZ model have triggered further investigations. In particular, it has been pointed out that **16**-plets localized at the fixed points with $\text{SO}(10)$ symmetry can be a very good starting point for the construction of realistic models [46]. These states are not split by the orbifold projection and all survive in the 4D spectrum, thereby providing complete matter generations of the SM. It is important that they form a GUT representation without having GUT symmetry in 4D.

The key idea of ‘local grand unification’ [52, 53, 47], which has been the guiding principle in the MSSM ‘MiniLandscape’ study [54, 55, 51], is that the structure of the MSSM can be explained in orbifolds with non-trivial gauge group topography as follows (cf. figure 10):

- (most) matter fields come from **16**-plets living at the fixed points with $\text{SO}(10)$ symmetry;
- $G_{\text{SM}} \subset \text{SO}(10)$ is the intersection of this ‘local $\text{SO}(10)$ ’ with other local gauge groups in $E_8 \times E_8$;
- Higgs fields live (mostly) in the bulk and appear as split multiplets.

$\text{SO}(10)$ local grand unification works only under certain conditions: one needs fixed points with $\text{SO}(10)$ symmetry admitting a massless **16**-plet. For example, \mathbb{Z}_3 and $\mathbb{Z}_2 \times \mathbb{Z}_2$ do not fall into this category. The simplest possibility would be a \mathbb{Z}_4 orbifold; however it turns out that there it is difficult to get three generations. The next-to-simplest possibility is then a \mathbb{Z}_6 orbifold. While the \mathbb{Z}_6 -I orbifold does not allow for a local $\text{SO}(10)$ and G_{SM} in 4D at the same time (since there is only one Wilson line, see [56]), it turns out that \mathbb{Z}_6 -II, utilized by KRZ, provides an excellent framework for the realization of local grand unification. It is also interesting that \mathbb{Z}_6 -II is favored for other reasons [57].³

The most obvious option to get 3 generations is to set the Wilson line in the $\text{SU}(3)$ torus to zero and use non-trivial Wilson lines in the $\text{SO}(4)$ torus. This guarantees triplication of families [46]. However, it turns out that all such models in \mathbb{Z}_6 -II suffer from the presence of chiral exotics

³Note that the nomenclature of [57] differs from ours: what is called \mathbb{Z}_6 -I corresponds to \mathbb{Z}_6 -II in these notes.

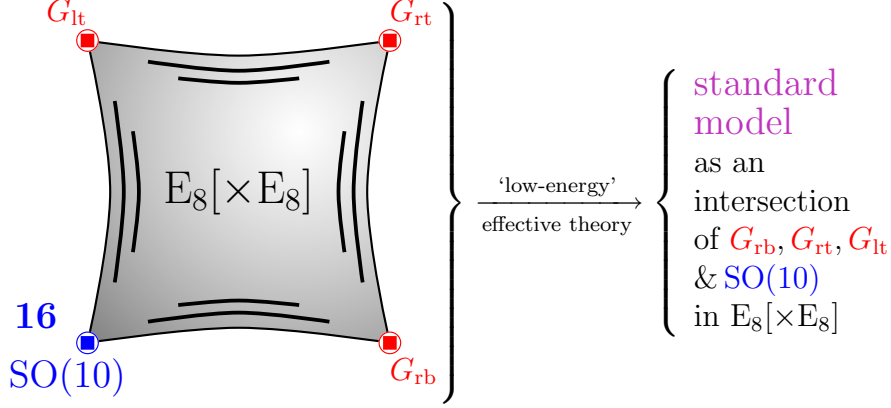


Figure 10: Local grand unification.

[47]. Similar considerations apply to all orbifolds up to order 6 inclusive. One is therefore led to consider models with 2 localized families where the third generation comes from the untwisted or higher twisted sectors. This implies, in particular, that the theory *requires* the first two and the third families to be fundamentally different. Needless to say, this result does not go against the data.

4.3 MSSM from heterotic orbifolds

The strategy of starting with two localized generations and requiring the third generation to come from somewhere else turns out to be successful. The first heterotic MSSM with this structure has been presented in [52, 47]. In this model, upon compactification one finds

$$\text{gauge group} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \times [\text{SU}(4) \times \text{SU}(2) \times \text{U}(1)^8] , \quad (31)$$

and

$$\text{spectrum} = 3 \times \text{generation} + \text{vector-like} , \quad (32)$$

where ‘vector-likeness’ is meant with respect to G_{SM} . The gauge group is broken further by requiring the Fayet-Iliopoulos (FI) D -term [58] to be cancelled, which implies non-zero vevs for some of the scalars. There are numerous possibilities to cancel the FI term while maintaining vanishing F -terms, corresponding to a large number of supersymmetric vacua. Most vacua are not phenomenologically interesting. What is, however, important in the model of [52, 47] is that it admits supersymmetric MSSM vacua where all the exotics become massive. To establish the decoupling of exotics, one has to prove that there are configurations where

- couplings of the form

$$x_i \bar{x}_j \prod_{\alpha} s_{i_{\alpha}} ,$$

where x_i and \bar{x}_j denote the exotics and $s_{i_{\alpha}}$ are standard model singlets, exist such that the exotics’ mass matrices have full rank;

- the s_{i_α} vevs are consistent with supersymmetry, i.e. F - and D -terms are zero.

To identify such configurations in practice is quite cumbersome (the interested reader is referred to [47, 51] for details). We note that giving supersymmetric vevs to the singlets has further benefits such as breaking the unwanted gauge factors (rank reduction), and the generation of effective Yukawa couplings, originating from higher-dimensional superpotential terms.

The main features of the MSSM vacua of the \mathbb{Z}_6 -II model [52, 47] are:

- ❶ exact MSSM spectrum, i.e.

$$3 \times \text{generation} + \text{Higgs} + \text{nothing} ,$$

where ‘nothing’ means a hidden sector;

- ❷ gauge group is G_{SM} (up to a hidden sector), with hypercharge being that of the usual $\text{SU}(5)$;
- ❸ due to the above features, the model is consistent with MSSM gauge coupling unification;
- ❹ the top Yukawa coupling is order 1, while all other couplings are small and generated *à la* Froggatt-Nielsen [59];
- ❺ there is an $\text{SU}(4)$ group in the hidden sector whose gauginos condense [60, 61, 62, 63] and can induce TeV soft masses

The model can also be viewed from the 6D orbifold GUT perspective which makes discussion of certain issues more tractable [47, 64].

Of course, this is not the only string construction with realistic features. For instance, in Calabi-Yau (CY) compactifications it is also possible to obtain models with the SM gauge group and matter content up to vector-like exotics [65]. Free fermionic constructions also provide models with realistic features [66, 67].⁴ A three generation Z' model based on the gauge group $G_{\text{SM}} \times \text{U}(1)_{B-L}$ has been constructed in [69, 70].⁵ A promising model on a Calabi-Yau manifold has also been reported in [71]. It has the exact MSSM spectrum directly after compactification, and enjoys non-trivial Yukawa couplings [72]. GUT-like constructions, leading to flipped $\text{SU}(5)$ models at the compactification scale, have been discussed in [73, 74]. Coming back to orbifold models, \mathbb{Z}_{12} -I also hosts interesting models, either with the flipped $\text{SU}(5)$ [75, 76] or the SM gauge group [77]. The \mathbb{Z}_3 orbifold, that has been under scrutiny for a long time, has been revived in the context of multi-Higgs extensions of the MSSM [78, 79]. More complicated orbifolds with non-factorizable tori and/or torsion are being actively studied [80, 81, 82].

4.4 Orbifold GUT limits

Having obtained string models with a simple geometric interpretation and realistic features, one might study orbifold GUT limits, which correspond to anisotropic compactifications where

⁴See however [68].

⁵In this model, $B - L$ cannot be broken without breaking supersymmetry. Whether a viable model can be constructed is not yet clear.

some radii are significantly larger than the others. This then leads to a ‘volume factor’ which can explain the differences between the 4D Planck scale, the string scale and the GUT scale M_{GUT} . M_{GUT} is, in this picture, essentially the inverse of the largest compactification radius, while other the radii are required to be smaller in order to stay in a regime where the description in terms of the weakly coupled heterotic strings can be applied [83, 84]. Various orbifold GUT limits have been derived in [45, 46, 47]. (In order to obtain a simple field-theoretic interpretation of the projection conditions it is convenient to transform the shift and Wilson lines to a particular form; see appendix A of [82].) Amazingly, it turns out that in many cases the orbifold GUT limits are consistent with gauge coupling unification [46, 47] in the following sense: the standard model gauge factors get combined into a simple bulk gauge group factor, such that at least the dominant threshold corrections (see [85, 86, 87] for the string calculations) are universal and do not spoil unification (see the discussion above figure 11 for an example).

4.5 Heterotic MiniLandscape

Having seen a particular example of the MSSM from the \mathbb{Z}_6 -II orbifold (subsection 4.3), it is imperative to ask whether more comparable models exist in this construction and whether they are frequent. This question has been answered affirmatively in [54, 55, 51], where the \mathbb{Z}_6 -II models with two localized **16**-plets of $\text{SO}(10)$ have been analyzed systematically.

In the \mathbb{Z}_6 -II orbifold, there are two gauge shifts that produce a local $\text{SO}(10)$ GUT with **16**-plets (cf. [88]),

$$\begin{aligned} V^{\text{SO}(10),1} &= \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0\right) \left(\frac{1}{3}, 0, 0, 0, 0, 0, 0, 0\right), \\ V^{\text{SO}(10),2} &= \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0, 0\right) \left(\frac{1}{6}, \frac{1}{6}, 0, 0, 0, 0, 0, 0\right). \end{aligned} \quad (33)$$

For each of these shifts, we follow the steps:

- ① Generate Wilson lines W_3 and W_2 .
- ② Identify “inequivalent” models.
- ③ Select models with $G_{\text{SM}} \subset \text{SU}(5) \subset \text{SO}(10)$.
- ④ Select models with three net **(3, 2)**.
- ⑤ Select models with non-anomalous $\text{U}(1)_Y \subset \text{SU}(5)$.
- ⑥ Select models with net 3 SM families + Higgses + vector-like.

It turns out that in these models almost 1 % has the MSSM spectrum plus vector-like exotics (table 1).

It is instructive to compare these results to other MSSM searches in the literature. In certain types of intersecting D-brane models, it was found that the probability of obtaining the SM gauge group and three generations of quarks and leptons, while allowing for chiral exotics, is less than 10^{-9} [89, 90]. The criterion which comes closest to the requirements imposed in [89, 90] is ④. We find that within our sample the corresponding probability is 6 %. In [91, 92], orientifolds of Gepner models were scanned for chiral MSSM matter spectra, and it was found that the

criterion	$V^{\text{SO}(10),1}$	$V^{\text{SO}(10),2}$
② inequivalent models with 2 Wilson lines	22,000	7,800
③ SM gauge group $\subset \text{SU}(5) \subset \text{SO}(10)$ (or E_6)	3563	1163
④ 3 net (3, 2)	1170	492
⑤ non-anomalous $\text{U}(1)_Y \subset \text{SU}(5)$	528	234
⑥ spectrum = 3 generations + vector-like	128	90

Table 1: Statistics of \mathbb{Z}_6 -II orbifolds based on the $\text{SO}(10)$ shifts (33) with two Wilson lines.

fraction of such models is 4×10^{-14} . These constructions contain the MSSM matter spectrum plus vector-like exotics. This is most similar to step ⑥ in our analysis where we find 218 models out of a total of 3×10^4 or 0.7 %. In comparison, approximately 0.6 % of our models have the MSSM spectrum at low energies with all vector-like exotics decoupling along D -flat directions. Note also that, in all of our models, hypercharge is normalized as in standard GUTs and thus consistent with gauge coupling unification. We learn from this comparison that the heterotic orbifolds with local $\text{SO}(10)$ structure are particularly “fertile” in producing the MSSM.

Having obtained a set of $\mathcal{O}(100)$ models with realistic features, one can (and should) study their properties. An interesting question is whether realistic features are correlated. In [55] the distribution of the scales of gaugino condensation in the models with an exact MSSM spectrum was studied. The scheme of gaugino condensation provides a natural explanation of the hierarchy between the Planck and supersymmetry breaking (or electroweak) scales. Gravity mediated SUSY breaking predicts an approximate relation

$$m_{3/2} \sim \frac{\Lambda^3}{M_{\text{P}}^2}, \quad (34)$$

where the gravitino mass $m_{3/2}$ sets the scale for the MSSM soft masses and Λ is the scale of the hidden sector strong dynamics. The distribution of Λ in our models shows that demanding realistic features in the observable sector leads to the preference for gaugino condensation at an intermediate scale. Therefore these models favour TeV-scale MSSM soft masses which provides a top-down motivation for low-energy supersymmetry [55].⁶

⁶We would also like to comment that in the context of Kähler stabilization [93, 94, 95], the hidden sector strong dynamics allows to fix the dilaton, as well as the other moduli, in particular the T -moduli [96, 97, 98], which parametrize the radii of the three tori. Of course, moduli stabilization is a complicated issue, and there are many different possibilities (see e.g. [99, 100]). In [96, 97] it was found that the T -moduli get fixed at order one values (in Planckian units), on the other hand, as briefly discussed in subsection 4.4, it appears desirable to obtain anisotropic compactifications. The results of [96, 97, 99] were obtained in the context of \mathbb{Z}_3 orbifolds without Wilson lines, where threshold corrections attain a particularly simple form [85, 86, 87]. It should be interesting to study whether in the \mathbb{Z}_6 -II orbifold with discrete Wilson lines there are hierarchies between the T -moduli vevs, leading to anisotropic compactifications.

4.6 A heterotic ‘benchmark model’

Somewhat surprisingly, the models of the MiniLandscape possess even more attractive features such as R -parity, the neutrino seesaw, etc. To be specific, let us focus on an explicit example: the vacuum of one of the MiniLandscape models, referred to as ‘1A’ in [51]. We would, however, like to emphasize that these features are shared by many other vacua of the MiniLandscape models.

R -parity. As in grand unification, it is possible to obtain R -parity as a \mathbb{Z}_2 subgroup of $U(1)_{B-L}$. An important difference however is that $U(1)_{B-L}$ is not embedded in $SO(10)$ and there can be SM singlets with even $B - L$ charge (not related to $\overline{\mathbf{126}}$ -plets of $SO(10)$). This facilitates the construction of MSSM vacua with an *exact* R -parity [54, 51], \mathbb{Z}_2^R . This is to be contrasted with [47] where R -parity was approximate, and to [72] where R -parity exists only at the classical level and Y_e and Y_d vanish at the same level. The model also does not suffer from the problem encountered in [77], where it was found that one can either decouple all exotics or have R -parity but not both.

μ -term. In model 1A, there is a vector-like pair of weak doublets, ϕ_u and ϕ_d , from the (third) untwisted sector. These correspond to the extra components of the 10D gauge bosons.⁷ They arise from an $SO(10)$ $\mathbf{10}$ -plet contained in the adjoint of E_8 . The pair $\phi_u \phi_d$ is neutral w.r.t. the selection rules, i.e. whenever a superpotential term $\psi_1^{n_1} \cdots \psi_r^{n_r}$ is allowed, also the term $\phi_u \phi_d \cdot \psi_1^{n_1} \cdots \psi_r^{n_r}$ is allowed. Further, it was found [51] that (at least at a given order) the global SUSY F -term equations are satisfied term by term,

$$\frac{\partial \mathcal{M}}{\partial s_i} = 0, \quad (35)$$

where \mathcal{M} denotes a monomial of SM singlets s_i entering the superpotential. One finds that also \mathcal{M} vanishes. That is, in supersymmetric vacua where the F -terms vanish term by term, the vev of the superpotential is zero. Further, the bilinear couplings involving ϕ_u or ϕ_d to any other $SU(2)$ doublet vanish because of the $G_{\text{SM}} \times \mathbb{Z}_2^R$ symmetries [51]. The $\phi_u \phi_d$ mass term can only be due to the above monomials \mathcal{M} . Since the latter is zero in the vacuum, the μ -term is zero and exactly one pair of Higgs doublets is massless (while the exotics are decoupled). In other words, demanding that

- $G_{\text{SM}} \times \mathbb{Z}_2^R$ be unbroken and
- F -terms vanish term by term

in this model leads to supersymmetric vacua with a suppressed μ -term. When SUSY gets broken, the μ -term of the order of the gravitino mass is generated. This constitutes a stringy solution to the MSSM μ problem.

⁷This property is shared by all models based on shift $V^{\text{SO}(10),1}$.

Gauge-top unification. The fact that the Higgs doublets stem from the untwisted sector has further important consequences. The left- and right-handed up-type quarks of the third generation are also untwisted and the corresponding interaction $\phi_u \bar{u} q$ is allowed by string selection rules. Furthermore, its strength is given by the gauge coupling since it stems from supergauge interactions in 10D. Thus the top Yukawa is predicted to be the same as the gauge coupling at the string scale.⁸ The other Yukawa couplings appear at higher orders and are therefore suppressed, similarly to the Froggatt-Nielsen picture [59]. The idea to relate the top Yukawa coupling to the gauge coupling is not new (see e.g. [101, 102]), however, the fact that this happens automatically in many models is remarkable.

Neutrino masses and see-saw. To discuss the neutrino masses in string-derived models one has first to clarify what a right-handed neutrino is. In supersymmetric MSSM vacua with R -parity, this question is answered quite easily: a right-handed neutrino is an R -parity odd G_{SM} singlet. In the model discussed so far, there are 49 such neutrinos. Further, as discussed more generally in [103], all ingredients of the see-saw [104] are present:

- the right-handed neutrino mass matrix M_ν has full rank and
- neutrino Yukawa couplings Y_ν exist

The resulting effective mass matrix for the light neutrinos,

$$m_\nu = v_u^2 \cdot Y_\nu^T M^{-1} Y_\nu, \quad (36)$$

where v_u denotes the vev of the up-type Higgs ϕ_u , has full rank. Thus, all light neutrinos have a small mass as a result of the seesaw mechanism. Due to the large number of neutrinos, the effective neutrino mass operator has many contributions such that neutrino masses are enhanced compared to the naive estimate $m_\nu^{\text{naive}} \sim v_u^2/M_{\text{GUT}}$. Let us also mention that meanwhile the many neutrino scenario has been analyzed in some detail. It has been found that the presence of many right-handed neutrinos helps ameliorate the tension between leptogenesis and supersymmetry [105, 106]. Furthermore, this scenario has important implications for supersymmetric lepton flavor violation and electric dipole moments [106].

Proton stability. Because of the exact R -parity, dimension four proton decay operators are absent. However, both $qqq\ell$ and $\bar{u}\bar{u}\bar{d}\bar{e}$ appear at order 6 in the SM singlets, and are also generated by integrating out the heavy exotics. This leads to effective dimension five operators mediating proton decay, the usual problem of 4D GUTs [9]. Forbidding such operators is likely to require further (perhaps discrete) symmetries [107], which is currently under investigation.

Non-Abelian discrete flavor symmetries. The models of the MiniLandscape exhibit a non-Abelian discrete flavor symmetry, D_4 , for the two light generations [45, 108]. This symmetry is only exact when the SM singlets have zero vevs, and is broken in realistic vacua. If the breaking is small, D_4 can be relevant to the (supersymmetric) flavor structure (cf. [109]).

⁸Again, this property to a large extent is shared by all models based on shift $V^{\text{SO}(10),1}$, in particular by the model of [52, 53].

Orbifold GUT limits. Last but not least, let us discuss orbifold GUT limits (cf. subsection 4.4) of this model. Consider the case where the radii of the $SO(4)$ torus, $R_{SO(4)}$, are significantly larger than the radii of the $SU(3)$ and G_2 tori, R_{other} . In the energy range $R_{SO(4)}^{-1} < E < R_{\text{other}}^{-1}$ the model can be described by an effective 6D theory with an $SU(6)$ bulk symmetry that gets broken to $SU(5)$ and $SU(4) \times SU(2)_L$ at the two left or the two right fixed points in figure 11, respectively (we are ignoring the second E_8 and $U(1)$ factors). The intersection of $SU(5)$ and $SU(4) \times SU(2)_L$ in $SU(6)$ is the SM gauge group G_{SM} . Since all standard model gauge group factors are contained in the bulk $SU(6)$, threshold corrections are universal, and the model is consistent with MSSM gauge coupling unification. Two of the three SM families reside at two equivalent $SU(5)$ fixed points while the third generation comes from the bulk (see figure 11). In order to understand why matter on the $SU(5)$ fixed points combines to complete generations, i.e. **16**-plets of $SO(10)$, one has to zoom into the fixed points and to resolve the underlying $SO(10)$ structure. By making the vertical direction in figure 11 small as well, one arrives at a setting that strongly resembles the Kawamura model [14, 15], except for the fact that the third family lives on the interval rather than on the boundary (cf. the analogous discussion in Ref. [64]).

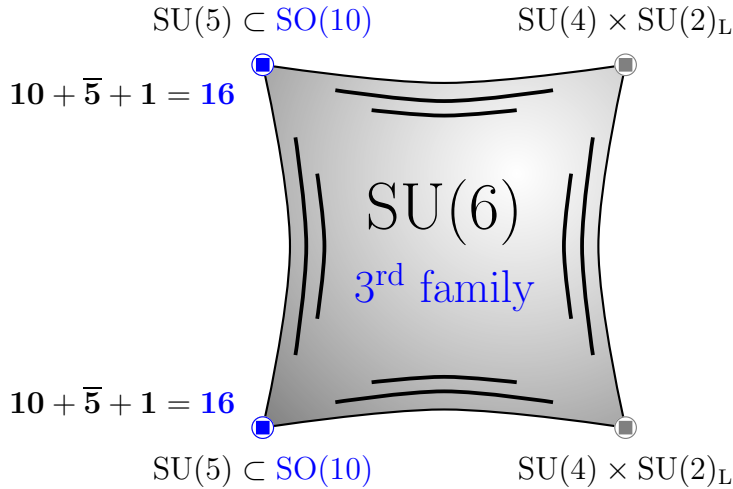


Figure 11: Orbifold GUT limit of model 1A of [51].

5 Summary

In summary, the scheme of local grand unification provides an attractive framework for connecting observations to fundamental physics. It shares many attractive features with conventional 4D GUTs while avoiding their most problematic aspects such as the doublet-triplet splitting problem and unrealistic fermion mass relations. In the context of string theory, the concept of local grand unification facilitates construction of phenomenologically attractive models. These are automatically anomaly-free, UV complete and also incorporate gravity. They also exhibit some surprising features including a novel stringy solution to the MSSM μ problem. Clearly, it is imperative to seek a more comprehensive understanding of these constructions.

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